Name:

SA402 · Dynamic and Stochastic Models Fall 2022 · Uhan

Quiz 9 – 12/7/2022

Problem	Weight	Score
1a	1	
1b	1	
2a	1	
2b	1	
2c	1	
3	1	
4a	1	
4b	1	
Total		/ 80

Instructions. You have the entire class period to complete this quiz. You may use your plebe-issue calculator and the provided list of formulas for birth-death processes and standard queueing models. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem 1. The Simplexville Post Office has 3 clerks on duty. Customers wait in a single queue and are served by the first available clerk, first-come first-served. The post office can only hold 30 customers total, including the 3 customers being served.

Potential customers arrive at the door of the post office a rate of 40 per hour. The interarrival times are exponentially distributed. When the post office has 20 or more customers total, potential customers balk with probability 1/2. In addition, any potential customers that arrive when the post office is full simply go elsewhere.

The average service time for a single clerk is 5 minutes per customer. The service times are exponentially distributed. Customers in the queue sometimes renege: the time a customer is willing to spend in the queue is exponentially distributed with a mean of 10 minutes.

Model this setting as a birth-death process by answering the following prompts.

a. Define the arrival rate in each state, in terms of the number of customers per hour.

b. Define the service rate in each state, in terms of the number of customers per hour.

Problem 2. Sean and Dan's Pizza Shop has a single queue for waiting customers and 2 cashiers. One of the cashiers is on duty at all times. The other cashier goes on duty whenever the queue of customers becomes too long; in particular, when the shop has 4 or more customers (including the customers being served). The shop can hold at most 8 customers total.

Suppose that customer arrivals are well modeled as a Poisson process with a rate of 20 customers per hour. The cashiers both work at a rate of 15 customers per hour, and the service times are well modeled as exponential random variables.

This setting can be modeled as a birth-death process with the following arrival and service rates (in customers per hour):

$$\lambda_i = \begin{cases} 20 & \text{for } i = 0, 1, \ldots, 7 \\ 0 & \text{for } i = 8, 9, \ldots \end{cases} \qquad \mu_i = \begin{cases} 15 & \text{for } i = 1, 2, 3 \\ 30 & \text{for } i = 4, 5, \ldots \end{cases}$$

The steady-state probabilities are:

$$\pi_0 = 0.09 \qquad \pi_1 = 0.13 \qquad \pi_2 = 0.17 \qquad \pi_3 = 0.22 \qquad \pi_4 = 0.15$$
$$\pi_5 = 0.10 \qquad \pi_6 = 0.07 \qquad \pi_7 = 0.04 \qquad \pi_8 = 0.03$$

a. Over the long run, what fraction of time are both cashiers busy?

b. Over the long run, what is the expected number of customers in the shop?

c. Over the long run, what is the expected time a customer spends in the shop?

Problem 3. Four Guys Burgers and Fries has 4 cashiers at its Simplexville location. Customers wait in a single queue and are served by the first available cashier, first-come first-served. The average service time is 2 minutes per customer, and customers arrive at a rate of 24 per hour. The interarrival times and the service times are best modeled with the exponential distribution. The Simplexville location is enormous and popular, so for all intents and purposes, the restaurant has infinite capacity and has an infinite number of possible customers.

Which standard queueing model fits this setting best?

Problem 4. You have been asked to take over the task of staffing maintenance teams at a facility that repairs and maintains SH-60 Seahawk helicopters. The helicopters arrive at the maintenance facility at a rate of 6 per week, and are processed on a first-come-first-served basis. It takes a maintenance team 1 day on average to repair 1 helicopter. The facility currently has 1 maintenance team working at any given time.

Model this system as an M/M/1 queue. For the problems below, use weeks as your time unit.

a. Over the long run, what is the fraction of time that there is a repair backlog (i.e., at least 1 helicopter awaiting repair)?

b. Suppose you find that $\pi_0 = 0.14$ (this may or may not match what you found in part a.). What is the expected time in queue (i.e., expected delay) for a helicopter that comes in for repair?